FUNDAMENTAL CONCEPTS OF WAVE-PARTICLE INTERACTIONS

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Abstract. Wave-pw[ic]c interactions are fundamental to all collisionless plasmas, from the instabilities generating the waves to the resonant interactions which cause pitch-angle scattering, particle energy diffusion and cross-f field diffusion of particles. These figures and text were developed for the 1995 Autumn College of Plasma Physics, held in Trieste, Italy, and were aired at explaining this phenomenon to a variable audience, from graduate plasma physics students to recent 1'11.1). fellows. So that the general AG U member may follow this discussion, 1 will discuss only the simplest concepts using undergraduate electricity and magnetism. 1 'or wave-pwliclc interactions involving electromagnetic waves, everything is explainable by the Lorentz force.

INTRODUCTION

Equation 1) is the Lorentz force in centimeter-gram-second (cgs) units. A particle with charge q moving with velocity $\vec{V}$ across a magnetic field 01 strength $\vec{B}_0$ experiences a force which is orthogonal to both V and $\vec{B}_0$. In the expression below, c is the velocity of light.

$$\vec{F}_L = \frac{q}{c}\, \vec{V} \times \vec{B}_0 \qquad (1)$$

Figure 1 illustrates this situation for a positively charged particle (e.g., a proton) moving exactly perpendicular to a uniform magnetic field, $\vec{B}_0$. $F_L = qV_\perp B_0/c$. $V_\perp$ is the velocity component perpendicular to $\vec{B}_0$ and in this case V, = V. The Lorentz force is balanced by the centrifugal force $mV_\perp^2/r$, where m is the particle mass and r is the particle gyroradius. Equating these two forces and solving for r, one gets r = 1', c/qB_0, where 1', is the particle perpendicular momentum. in this uniform magnetic field, the particle exhibits a circular motion about the magnetic field where the frequency of motion, $2\pi r/V_\perp$. is equal to mc/q B_0, the cyclotron (or Larmor) frequency, Ω, of the charged particle.

Figure 2 illustrates the concept of a particle pitch angle. For this particular example, the particle charge is positive (positive ion). In a uniform magnetic field, the angle that the instantaneous particle velocity makes relative to the magnetic field vector is constant and is called the pitch angle. The particle velocity vector can be broken down into two orthogonal components, one parallel to $\vec{B}_0$, $V_\parallel$, and the other perpendicular to $\vec{B}_0$,$V_\perp$ (equation 2). Since there are no forces exerted on the particle in the parallel direction, the particle moves unimpeded with a constant velocity $V_\parallel$ along $\vec{B}_0$. There is also a cyclotron motion associated with the $v_\perp$ component. Although the direction changes, the magnitude of $V_\perp$ remains unchanged. A positively charged particle thus moves in a left-hand spiral motion along the magnetic field. This handedness is important for resonant interactions, as we will illustrate later. Positive ions gyrate in a left-h;md sense relative to $\vec{B}_0$, independent of whether they are moving along $\vec{B}_0$ or antiparallel to $\vec{B}_0$.

$$\vec{V} = \vec{V}_{\parallel} + \vec{V}_{\perp} \tag{2}$$

Because electrons (anions, and negative ions) have negative charge, the $\vec{V} \times \vec{B}$ Lorentz force is oppositely directed from that of positively charged ions. Thus, electrons gyrate about the magnetic field in a right-hand corkscrew sense, opposite to that shown in Figure 2.

If there is a strong magnetic field gradient, the particles can be "mirrored", or reversed in direction, by the Lorentz force. We show a particle at its mirror point in Figure 3 to illustrate this point. Although the Figure indicates a one dimensional gradient with a positive sense to the right, the reader should imagine this to be a two-dimensional gradient where similar field line convergences occur into and out of the paper as well. At the moment in time where the particle is being mirrored, all of its velocity (energy) is in the perpendicular (to the field) frame. Due to the convergence of the magnetic field lines, the Lorentz force has a component toward the left, leading to particle acceleration in a direction opposite to the gradient, and thus "reflection".

Since the Lorentz force operates in a direction orthogonal to the velocity vector, there s no work done. The total energy of the particle remains constant. Squaring equations (2 and multiplying by 1/2m, we get:

$$E_T = 1/2mV^2 = 1/2mV_{\|}^2 + 1/2\ mV_{\perp}^2 = E_{\|} + E_{\perp} \quad (3)$$

For a particle moving in a constant magnetic field, the parallel and perpendicular energies $E_{\|}$ and $E_{\perp}$ are each constant values. However, for a particle moving from left to right in a magnetic field gradient, as shown in Figure 3, $E_{\|}$ decreases as $E_{\perp}$ increases, keeping E_T constant. The mirror point is reached when $E_{\perp} = E_T$. The particle then starts to move to the left with $E_{\|}$ increasing (and $E_{\perp}$ decreasing).

A magnetic bottle has positive gradients at both ends of the field lines as is shown in Figure 4. Particles with large pitch angles are "trapped" and will have two mirror points. The particles will bounce back and forth between its mirror points. However, particles that have 0° pitch angles or angles close to 0° will mirror at only extremely high field strengths and will thus escape out the ends of the bottle.

If one bends the lines of force, to a shape of a dipole field (Figure 4, bottom), we have the general shape of planetary magnetospheric fields. Particle radiation, such as the Van Allen radiation belts, are trapped on these fields. The particles gyrate about the magnetic fields and also bounce back and forth between their mirror points. How one gets particles on these trapped orbits is another problem. One way is to have cosmic ray particles have nuclear interactions with upper atmosphere atoms and molecules. Neutrons produced in these interactions decay into protons and electrons within the magnetosphere populating the belts. These are called CRAND (cosmic ray albedo neutron decay) particles.

The "loss cone" is the cone of pitch angles within which particles in this cone are lost to the upper atmosphere. The particle mirror points are deep in the atmosphere and the particles thus lose their energy by collisions with atmospheric/ionospheric atoms and molecules and do not 1'1'1111"11 to the magnetosphere. Thus, the magnetospheric equatorial phase space (pitch angle) distribution has signatures that looks like Figure 5a.

The size of the 10ss cone can be calculated by assuming constancy of the first adiabatic invariant μ. μ is equal to $E_\perp/B_0$. For the dipole field shown in Figure 5b, wc calculate the value below. As previously mentioned, at the mirror point, the particle's perpendicular kinetic energy $E_\perp$ is equal to the total kinetic energy, E_T. Thus we can write μ as E_T/B_{Mirror} at the mirror point. At the equator, μ is equal to $E_\perp/B_{eq}$. Equating these two values we have $E_T/B_{Mirror} = E_\perp/B_{eq}$. We rearrange this as $E_\perp/E_T = B_{eq}/B_{Mirror}$. However, from previous discussions, we know that $E_\perp / E_T = \frac{1}{2} mV_\perp^2 / \frac{1}{2} mV^2 = \sin^2 \alpha$.

'1'1)11s, for the loss cone wc have:

$$\sin^2 \alpha_o = B_{eq}/B_{Mirror} \tag{4}$$

The values for B_{eq} and B_{Mirror} can be calculated assuming a dipole field dependence with distance, B/r^3 = constant. At the Earth, the surface equatorial field is approximately ().3 Gauss. Thus for any dipole field line, the 10ss cone can be easily calculated. For any particle with pitch angles at the equator with $\alpha < \alpha_o$, such that the height 01 the mirror point is within the upper atmosphere, the particles are lost by collisions with neutrals. in the expression, α_o is the pitch angle at the edge of the 10ss cone.

The Earth's field is not a pure dipole. There are variations in the local surface field strengths. One area, called the Brazilian anomaly (previously talk'd the South Atlantic Anomaly, but this magnetic region has recently drifted inland) is a region of low magnetic field strengths. In this region, the magnetic fields are weaker and charged particles mirror at lower altitudes and are lost through collisions. Satellites passing through this region must endure higher radiation doses.

Previously, we showed that charged particles have a circular (cyclotron) motion about the ambient magnetic field (gyromotion) plus a translational motion. Under certain conditions, electromagnetic waves, can resonate with the particles. Below is the cyclotron resonant condition:

$$\omega - \ddot{k} \cdot \ddot{V} = n\Omega \qquad (5)$$

In expression (5), ω and $\ddot{k}$ are the wave frequency and k vector, and n is an integer equal to 0, ± 1, ± 2, ... The above expression mathematically shows that resonance occurs when the wave is Doppler-shifted to the particle cyclotron frequency by the factor $\ddot{k}$ V, the relative motion between the particle and the wave.

For illustrative purposes, we will only describe the first order resonance (n = 1) and will assume that the wave $\dot{k}$ is oriented along the magnetic field direction.

Thus (5) simplifies to:

$$\omega - \ddot{k}_{\|} V_{\|} = \Omega \qquad (6)$$

If the frequency of the wave and the local gyrofrequency of the particle are known, then the particle resonance energy, can be calculated. There are several simple steps to get from equation (6) to (7).

$$E_{\|} = 1/2\, m V_{\|}^2 = 1/2\, m V_{ph}^2 (1 - \Omega/\omega)^2 \qquad (7)$$

In the above expression, $\omega/k = V_{ph}$, the wave phase speed. For the case in which the resonant waves are at frequencies less than the ion cyclotron frequency, the wave-phase speed can be approximated by the local Alfvén speed V_A. $V_A = [B^2/8\pi\rho]^{1/2}$, where ρ is the ambient plasma density. This approximation is often used for pitch angle diffusion rate calculations.

1 'igure 6 illustrates the spatial variation of the wave (perturbation) magnetic vector as a function of distance along the magnetic field. Here wc will illustrate circularly-p olarized, parallel-propa; ating electromagnetic waves. '1'here are two basic types of polarization, right-and-left ianded. Elliptical or linear polarizations are combinations 01 these two polarizations.

In a magnetized plasma, left-hand polarized waves can exist at frequencies up to the ion Cyclotron frequency. At the high end of the freq uency range, this mode is called an ion cyclotron wave. At low frequencies , this mode maps into the Alfvén mode branch. Right-hand waves can exist up to the electron cyclotron frequency. These waves are dispersive (in this case, higher frequencies have higher phase velocities), and when (hey travel any substantial distances, the highest frequencies arrive first. 1 ightning-generated electromagnetic noise traveling within a plasma "dud" (:lficl(l-aligned density enhancement or depletion with $\Delta\rho/\rho \geq 5\%$) from one magnetospheric hemisphere to the other, ends up having a whistling sound, thus the name "whistler mode". At low, or MHD frequencies, this wave maps into the magnetosonic mode.

The polarization of waves is defined by the sense of rotation about the ambient magnetic field. This is independent of the direction of propagation. The same is true for positive ion (left-hand) and electron (right-hand) gyromotions about the magnetic field.

The normal cyclotron resonance between waves and charged particles are pictorially shown in Figure 7 For this resonant interaction, the waves and particles propagate towards each other. Left-hand positive ions interact with left-handed waves, and correspondingly right-hand electrons interact with right-hand waves. Since the waves and particles approach each other, $\vec{k} \cdot \vec{V}$ has a negative sign. Thus the Doppler shift term $(- \vec{k} \cdot \vec{V})$ in equation (5) is a posi ive one. The relative motion of the wave and particle Doppler-shifts the wave frequency, ω, up to the particle cyclotron frequency, Ω.

One plasma instability generating these waves in planetary magnetospheres is the "loss-cone instability". Th is i ty occurs when conditions $T_{\perp} / T_{\parallel} >$ exist. $T_{\parallel}$ $T_{\perp}$ is the ion or electron temperature parallel (pe pendicular) to $\vec{B}_0$ Electron loss cone instabili ies generate whistler-mode emissions descriptively called auroral zone "chorus" and "plasmaspheric hiss" because of the sounds they make when p ayed through a speaker he waves in the outer magnetosphere do not bounce several times ike lightning whistlers, and frequency-time structures are due to generation mechanisms).

There is another type of resonance called anomalous cyclotron resonant interactions. This is shown for the case of positive ions in Figure 8 Posit ve ions interact with right-hand waves. They do so by overtaking the waves ($V_{\parallel} > V_{ph}$ such that the ions sense the waves as left-hand polarized Because the left-hand ion i teracts wit a ight-hand wave, this interaction is called "anomalous rom the expression in the resonance condition, the Doppler shift decreases the wave frequency to that of the cyclotron frequency. xamples of the instability generating such waves is the ion beam instability in planetary foreshocks ar d ion pickup around comets. The ion beam generates right-hand magnetosonic waves. In the foreshock case, solar wind ions reflect from the bow shock, and stream outward into the solar wind These particles are typically 5 keV n kinetic energy ligher energy particles from the magnetosheath extend this energy to -40 keV. In the cometary case, neutral molecules/atoms sublimate from the nucleus as the comet approaches the Sun. This neutral cloud can be - 10(' km in radius. The photoionization and charge exchange of cometary 1170 group neutrals (1 I$_2$0, 011, ()) lead to the formation of a "beam" in the solar wind frame. For the instability, the typical kinetic energy of the ions relative to the solar wind plasma is 30-60" keV.

The same anomalous cyclotron resonant interactions occur between electrons and left-hand mode waves. 1 Iowever, since the left-hand waves are at frequencies below the ion cyclotron frequency (a value far below the electron cyclotron frequency), resonant electrons are typically relativistic in energies ($E_{||}$ > 1 MeV) for this interaction. Even so, it is speculated that such an instability is occurring upstream of the Jovian magnetosphere perhaps due to leakage of Jovian radiation belt electrons.

The actual physical mechanism for particle pitch angle scattering due to electromagnetic waves is the Lorentz force. This is illustrated pictorially in Figure 9 for positive ions. At cyclotron resonance, the particle experiences the wave magnetic field B_w gyrating in phase with the particle. For ease of visualization, we separate particle $V_\perp$ and $V_{||}$ components. Clearly the resonant interaction of particles with arbitrary pitch angles will be a combination of the two. In panel 9a), we show the case when the interaction is through $V_\perp$. Since a constant B_w is imposed on the particle, the Lorentz force is in the $\vec{B}_0$ direction. If the particle is propagating towards the right, the pitch angle will be decreased, and if the particle is traveling to the left, it will be increased. However, we have arbitrarily chosen the $\vec{B}_w$ to be in the upward direction in the Figure. If the relative phase between the wave and particle were shifted by 180° such that $\vec{B}_w$ was pointed downward, all of the results stated previously would be reversed.

Panel (9b) shows the particle interaction due to the parallel component of particle velocity. Here the Lorentz force is in a direction opposite to that of the gyromotion of the left-hand ion. Therefore, the interaction decreases $V_\perp(E_\perp)$ and decreases the pitch angle of the particle. If the phase of the wave was 180° different, such that $\vec{B}_w$ was directed downward, $\vec{F}_\perp$ would accelerate the particle in $E_\perp$, and the pitch angle would be increased.

Resonant wave-]xw[ic]c interactions occur on time scales small compared to the cyclotron period, thus the first adiabatic invariant μ is not conserved. The total energy Of the particle is not conserved. To see this, we examine the energy of the particle in the rest frame of the wave. 1 lere the total energy is constant:

$$1/2mV_\perp^2 + 1/2m\,(V_\parallel - V_{ph})^2 = \text{constant} \qquad (8)$$

Assuming small perturbations in $V_\perp$ and $V_\parallel$, we get $mV_\perp\Delta V_\perp + mV_\parallel\Delta V_\parallel - mV_{ph}\Delta V_\parallel = 0$. However, earlier we showed that $E = 1/2mV_\perp^2 + 1/2\,mV_\parallel^2$ or $\Delta E = mV_\perp\Delta V_\perp + mV_\parallel\Delta V_\parallel$. Substituting the above into expression (8), we have:

$$\Delta E = mV_{ph}\Delta V_\parallel \qquad (9)$$

So we find that the particle energy changes, based on the sign of $V_\parallel$. Particles that increase $V_\parallel$ through resonant interactions increase energy and absorb wave energy, and those that decrease $V_\parallel$ lead to the generation of wave energy. In general, if one starts with a highly anisotropic pitch angle distribution (say $T_\perp >> T_\parallel$), as the waves stochastically scatter the particles and "fill" the loss cone, the energy of the particles overall decreases, with the concomitant increase in wave energy.

For waves with electric field amplitudes, E_w, the particle's perpendicular kinetic energy increases or decreases depending on the phase of the wave with respect to the particle. The situation for increased $E_\perp$ is shown in Figure 10. Analogous arguments can be made for wave-particle interactions due to electrostatic waves or the electric component of electromagnetic waves. We have left the details of these interactions as exercises for the reader.

The overall particle pitch angle scattering rates due to electromagnetic or electrostatic waves have been defined by Kennel and Petschek (1966), and have empirically been shown to be quite valid for the rate of scattering of electrons in the outer magnetosphere. From the derivation of their equation (14), we have $\tan\alpha = V_\parallel/V_\perp$ and for large pitch angle particles where $V_\perp \cong V$,

$$\Delta\alpha = -\Delta V_\parallel / V_\perp \qquad (10)$$

The amount of acceleration that the particle receives is the Lorentz force divided by mass times the time (Δt) that the wave and particle stay in resonance. This can be expressed as:

$$\Delta\alpha = \frac{eV_\perp}{cm} B_w \Delta t \frac{1}{V_\perp} = \frac{B_w}{B} \Omega \Delta t \qquad (11)$$

The pitch angle diffusion rate is thus:

$$D = \frac{(\Delta\alpha)^2}{2\Delta t} \cong \frac{\Omega^2}{2}\left(\frac{B_w}{B}\right)^2 \Delta t \qquad (12)$$

The time At is the time a particle $\Delta k/2$ out of resonance changes its phase by one radius, or $\Delta t \cong 2/\Delta k V_\parallel$.

We now get:

$$) \approx \Omega \frac{B_w^2/\Delta k}{B} \frac{\Omega}{V\cos\alpha} = \Omega \frac{B_w^2/\Delta k}{B^2} \frac{k}{\cos\alpha} \tag{13}$$

Agai assuming large pitch particles,

$$) \approx \Omega \left(\frac{B_w^*}{B} \right)^2 \eta \qquad 4)$$

Where B_w^{*2} is the wave power spectral density near esonance, and η is the fractional amount of time that the particles are in resonance witl the waves.

The Pederson mobility of particles i the direction perpendicular to $\vec{B}$ (Schultz and anzerot i 1974) is:

$$\mu_\perp = (c/B)\Omega\tau_{eff}/[1+(\Omega\tau_{eff})^2 \qquad (5$$

n the above expression τ_{eff} is the effective ime between wave-particle "collisions".

The maximum cross-field diffusion occurs when the particles are scattered at a rate equal tc their gyrofrequencies, or $\tau_{eff}^{-1} \approx eB/mc$ (Bohm diffusion). A spatial diffusion coefficient derived by Rose and Clark 96 is:

$$D = \langle \Delta X \rangle^2 / 2\Delta\tau = \left(mV_\perp^2 / 2e \right) \mu_\perp \qquad 6)$$

For Bohm 949) diffusion,

$$D_\perp \approx E_\perp c/2eB \approx D_{max} \tag{17}$$

If we substitute the expression for $D_{\alpha\alpha}$ into the Kennel-Petschek (1966) expression (their equation 14), for conditions where $\Omega\tau >> 1$ and $\tau_{eff} \approx 1/D_{\alpha\alpha}$ we can approximate the cross-field diffusion rate (Tsurutani and Thorne, 1982) by:

$$D_{\perp,\parallel} \approx \frac{E_\perp}{e}\frac{c}{B}\Omega\tau_{eff} \approx 2\left(B_w^*/B\right)^2 D_{max} \tag{18}$$

Similarly for electrostatic waves, we get

$$D_{\perp,\varepsilon_w^*} \approx 2\left(E_w/B\right)^2\left(c/v\right)^2 D_{max} \tag{19}$$

Figure 11 shows the process of cross-f -field diffusion due to resonant wave-particle interactions. $\vec{B}_0$ is the original guiding center. After pitch angle scattering, $\vec{B}_0'$ is the new guiding center. The particle has "diffused" across the magnetic field line.

FINAL COMMENTS

We have tried to give simple explanations with illustrations to explain the fundamentals of wave-particle interactions. Clearly, more complex interactions and second-order effects which indeed are present, have not been included in this basic description.

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Figure Captions

Figure 1, The Lorentz force and a positively charged particle gyromotion in a uniform magnetic field.

Figure 2. The "pitch" angle of a positively charged particle.

Figure 3. A schematic illustrating the mirror force.

Figure 4, A magnetic bottle for plasmas.

Figure 5. The loss cone (a) and auroras (b) associated with particle pitch angle scattering into the 10ss cone.

Figure 6. Left- hand and right-hand circularly polarized electromagnet ic waves.

Figure 7. Normal first-order cyclotron resonance between electromagnetic circularly polarized waves and charged particles.

Figure 8. Anomalous cyclotron resonance between electromagnetic circularly polarized waves and charged particles.

Figure 9. Pitch angle scattering by resonant electromagnetic waves.

Figure 10. Pitch angle scattering by resonant electrostatic waves.

Figure 11. Particle cross-field diffusion by a resonant interaction with waves.